Continuous Variables Nonlocality without Entanglement

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The most celebrated manifestation of the Nonlocality of Quantum Mechanics arise from entangled states, that is states of a compound system that admit no description in terms of states of the constituent parts. In view of this, one might expect that if the states of a quantum system were limited to a set of orthogonal product states, the system would not exhibit any nonlocality. In 1998, Bennett et. al. showed that it was not the case by exhibiting a set of 9 bipartite orthogonal product states, each party holding a qutrit, that can be reliably distinguished by a joint measurement on the entire system, but not by any sequence of Local Measurements on the parts and Classical Communications (LOCC). To set the idea, we can imagine the following game; a preparator randomly chooses a number between 1 and 9, accordingly prepares the corresponding quantum state and send one qutrit to Alice located at A and the other one to Bob located at B. He then asks Alice and Bob to determine which state of the set they were given, i.e. to find the value of the label $i$. If quantum communications are allowed, Alice and Bob can find a joint measurement that will determine the label with certainty as the states are orthogonal. On the other hand, Bennett et.al. have shown that if now Alice and Bob are restricted to Local Operations and Classical Communications, they will never be able to perfectly accomplish their task. More specifically, it is shown that the mutual Information Alice and Bob can extract from the set through LOCC will always be upper bounded by a quantity strictly less than $\log_2 9$ (the mutual information for the optimal joint measurement) and they manage to numerically calculate this value. So it is as if the product set reveals himself to be richer when considered jointly then when considered locally and in this sense exhibits true nonlocality.

There exists another known example of a tripartite set, each party holding a qubit, that exhibits Nonlocality without Entanglement:

$$\begin{align*}
|\Psi_1\rangle &= |0\rangle_a \otimes |0\rangle_b \otimes |0\rangle_c \\
|\Psi_2\rangle &= |0\rangle_a \otimes |1\rangle_b \otimes |0 - 1\rangle_c \\
|\Psi_3\rangle &= |0\rangle_a \otimes |1\rangle_b \otimes |0 + 1\rangle_c \\
|\Psi_4\rangle &= |1\rangle_a \otimes |1\rangle_b \otimes |1\rangle_c \\
|\Psi_5\rangle &= |1\rangle_a \otimes |0 - 1\rangle_b \otimes |0\rangle_c \\
|\Psi_6\rangle &= |1\rangle_a \otimes |0 + 1\rangle_b \otimes |0\rangle_c \\
|\Psi_7\rangle &= |0 - 1\rangle_a \otimes |0\rangle_b \otimes |1\rangle_c \\
|\Psi_8\rangle &= |0 + 1\rangle_a \otimes |0\rangle_b \otimes |1\rangle_c
\end{align*}$$

In this work we have realized that the latter set could be constructed from the computational basis using a simple quantum circuit. This circuit allowed us to get a strong insight into the origin of the phenomenon, and identify the required ingredients to construct set exhibiting Nonlocality without Entanglement. The comprehension we gained from the circuit helped us design a general method to construct such sets with qudits of any dimension. We generalized on step further and gave the first example of Nonlocality without Entanglement with Continuous Variables using squeezed states.